Construction of optimum grids of implicit surfacesby the method of normalization

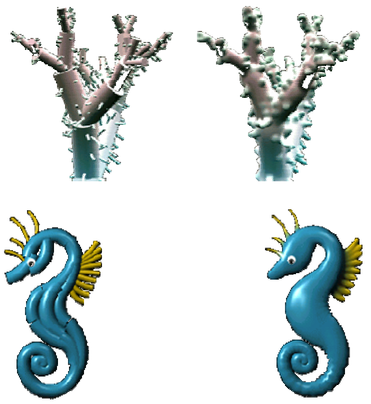
A.A. Litvinenko

NSTU, Department of Applied Mathematic and Computer Science
630092, Novosibirsk, K.Marks, 20, Russia

Abstract – **In this paper we present the technique for construction of optimum grids of implicit surface. The original algorithm is described.**
Keywords: implicit function, implicit surface, mesh optimization

I. INTRODUCTION

With evolution of hardware the technique of modeling with the help of implicit functions is used all more actively ([1], [2], [3], 0). Today there are the graphic systems, which provide for the user various tools of modeling; among them there are also implicit surfaces. They are successfully used, for example, in design of smooth objects of arbitrary topology, in animation of physical-based objects.



«Coral tree» and «Hippocampus», represented in A. Sherstyuk [3]
Figure 1

«Hippocampus» is constructed using the technique described in this work
Figure 2

II. IMPILICIT SUFACE

The implicit surface is mathematically defined in space as the set of points $p=(x,y,z)$, satisfying the equation $f(p)=0$.

Convenience of such representation consists that any operations with implicit surfaces, such as addition, subtraction, union, intersection etc., are reduced to mathematical calculations with the functions assigning these surfaces.

In the work, implicit surface S is represented as an isosurface of level T in scalar field $F(p)$.

$$S=\{p|F(p)-T=0\}$$

The function of complex field F is defined, as the addition of fields of primitives:

$$F(p)=\sum_{i=1}^{N} f_i(p)$$

As the primitives in the work, the convolution surfaces are used. The convolution surface is the implicit surface based on field function *f(p)*, obtained as space convolution of two scalar functions *g(p)* and *h(p).* Geometrical function *g(p)* gives a spatial distribution potentials-radiates, *h(p)* is a kernel of convolution which specifies damping of the potential produced by a radiant.

$$f(p)=g(p)\circ h(p)=\int_{R^3} g(r)\cdot h(p-r)\,dr$$

The kernel of convolution *h(p)* should be represented by the function which value sharply decreases depending on distance. In the work, the function of Koshy is used.

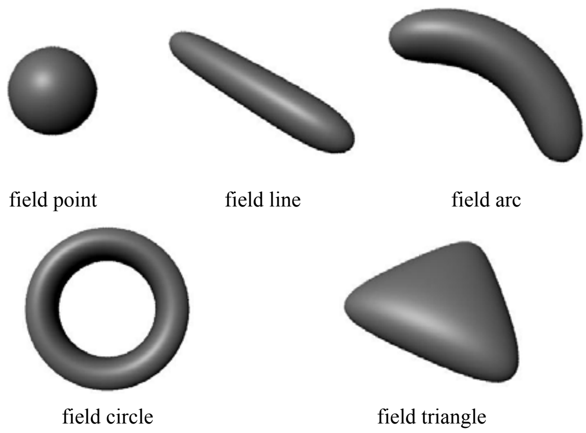
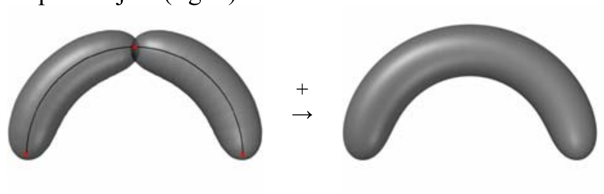


Figure 3

As primitives in the work, fields of a point, a line, an arc, a circle, a triangle are defined (fig. 3). Usage of such functions was offered in A. Sherstyuk's thesis [2].

Advantages of such representation of implicit functions following:

1. At modeling with the help of primitives the features of skeletal modeling are appeared.
2. It is easy to receive smooth concatenations in the complex object (fig. 4).



two primitives of field arc complex field
Figure 4

III. THE METHODS OF POLIGONIZATION OF IMPLICIT SURFACES

For visualization of an implicit surface may be used the method of ray tracing, but it is laborious method and its outcome may be found not suitable for the further operation.

At interactive modeling, for instance, for shading or texturing and other operations, is required to conduct poligonization of implicit surfaces and interactiveness superimposes the serious requirements on velocity of poligonization algorithms.

One of the most popular poligonization methods uses technique of a space partition. The space is divided into cells (for example, cubic) among which are being searched enveloping a surface from two sides. Further, from these cells are created triangulation (is more detailed see. 0)

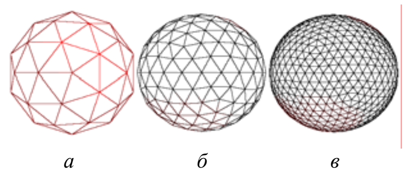
In the work we have implemented the method of marching triangle, which have offered E. Galin and S. Akkouche [1]. This method directly generates a mesh of an implicit surface with almost everywhere the equivalent triangle.

We have developed a fast method of build-up of grids of the complex implicit surfaces on the basis of grids of primitives. The idea of a method is following: at build-up of a grid of union of implicit surfaces is maximally fully to use the grids available on the previous step. More good outcomes managed to be achieved, using the grids available on the previous step, and rebuilding a grid by method of marching triangle in particular places.

In earlier implemented methods we have seen a considerable shortage. During build-up of a polygon grid the information on a local structure of a surface (curvature of a surface) is not being taken into account. In this work the method that takes into account curvature of a surface during poligonization is presented.

The method of normalization is a method of moving of a grid from a single orb on implicit surface *S*.

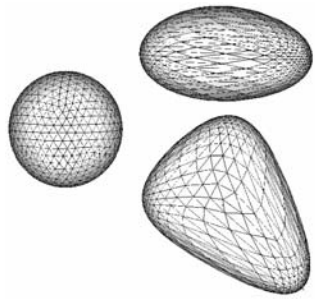
At the first step the template of a polygon grid on a single orb is selected. Approximating of an orb, with the help of a regular icosahedron is used. For deriving more small-sized approximating, average points of rib of triangle are taken, and their central projections on an orb are found. Junctions of the obtained points divide each triangle into more small-sized four triangles (fig. 5).



optimal grids of an orb consisting of a) 80, b) 320, and c) 1280 triangles
Figure 5

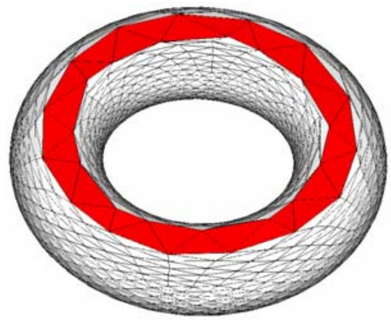
At moving of vertex *A* of a grid of an orb on surface *F*, the task of searching of a point $\phi(A)$ on surface *F*, with a known value of a gradient is being solved. At a numerical solution of the given task, curvature of a surface also is used.

Projection of an orb to ovaloids (surface of the positive curvature) is the univocal. For an ovaloid, the task of construction of an optimum grid is easily solved (see fig. 6). Here each point of an orb is once designed on an implicit surface.



Moving of a grid from a single orb on a field triangle
Figure 6

The arbitrary surface may contain areas of the positive and negative curvature; therefore projection of an orb to such surface is not the univocal. We shall consider a field of a circle (fig. 7).



Moving of a grid from a single orb on a field circle
Figure 7

Here already each vertex of grid of an orb two times is projected on an implicit surface. And area filled on a figure 7 generally corresponds to one vertex of grid on an orb. Such areas are generally characterized by that one of principal curvatures on them is enough small, i.e. $\min\left(k_1^S, k_2^S\right) < \varepsilon$, where k_1^S, k_2^S - principal curvatures/

The algorithm for moving of a grid from single orb T on arbitrary implicit surface S is following:

1. Find the point $A' \in S$, in which condition $\min\left(k_1^S, k_2^S\right) > \varepsilon$ is performed.

2. For the point $A' \in S$ find the vertex $A \in T$, in which $\operatorname{grad} S\left(A'\right) \approx \operatorname{normal} T\left(A\right)$.

3. For arbitrary vertex B, C formating a triangle $\triangle ABC$ with the vertex A at the orb grid find projection $\phi(\mathrm{A}), \phi(B), \phi(C) \in S$ starting with point $A' \in S$. During find the projections, it is necessary that condition $\min\left(k_1^S, k_2^S\right) > \varepsilon$ is performed. If all three projection $\phi(\mathrm{A}), \phi(B), \phi(C) \in S$ are not found goto step 1. If found, construct $\triangle\phi(A)\phi(B)\phi(C)$ – this is started triangle for building patch of the surface (fig 8).

4. Considering triangles the contiguous with already constructed, extend a patch of a surface. Each time project a new vertex on implicit surface. During searching projections keep up, that a requirement $\min\left(k_1^S, k_2^S\right) > \varepsilon$ is performed. During construction of triangles it is fixed the information about already projected vertices of a grid of an orb for not allow undesired overlay.

5. If addition of a new triangle to a patch of an implicit surface is not success because of breaking a requirement $\min\left(k_1^S, k_2^S\right) > \varepsilon$. Signifies it is necessary to try to cross through area of an implicit surface where $\min\left(k_1^S, k_2^S\right) < \varepsilon$.

6. Select one of boundary edges and move along one of principal a direction appropriate main to curvatures. When came in area where $\min\left(k_1^S, k_2^S\right) > \varepsilon$, check up, whether the patch of a surface in this area is constructed already. If yet it is not constructed, transfer to a step 1. If it is already constructed, join two patch of a surface by triangle and again fulfill a step 6.

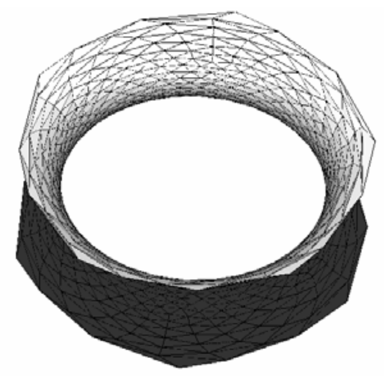


Figure 8

Thus, all patches of a surface where $\min\left(k_1^S, k_2^S\right) > \varepsilon$ will be constructed (fig. 9) and the hole between patches is sealed (fig 10, 7).

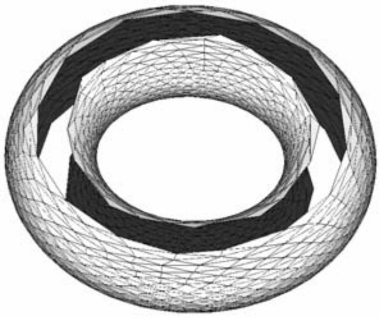


Figure 9

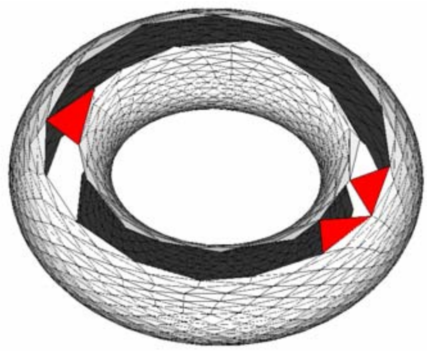


Figure 10

IV. IMPLEMENTATION

Implementation of described methods is being advanced in two directions. First: an application is being created in Microsoft Visual Studio. NET written on Microsoft Visual C++, where necessary data structures are designed and all circumscribed methods (for visualization graphics OpenGL library is used) are implemented. Second: a plug-in is being created for a system of computer graphics Maya of version 6.0, using API Maya, where with usage of interior data structures Maya all circumscribed methods are implemented.

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